Adiabatic rocking ratchets: Optimum-performance regimes

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We analyze work and efficiency for an adiabatic rocking ratchet working under three operating regimes: maximum efficiency, maximum work, and a third one which represents a compromise between them. For all of these regimes the application of very concrete loads and external amplitudes is found necessary in order to obtain the maximum possible values of both efficiency and work. The reported results could be valuable to design efficient Brownian motors and compare their operation under different working regimes.

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I. INTRODUCTION

In the last decades finite-time thermodynamics (FTT) has provided an extended framework to analyze more realistic upper bounds for the performance of real, irreversible heat devices beyond the limits imposed by the classical thermodynamics [1,2]. A central point in FTT is the optimization of a given heat device model constrained by finite-size and/or finite-time processes. To get this, an optimization objective is proposed and making use of the optimal control theory or variational principles one finds the thermodynamic conditions fitting such optimization criterion. In this context some of us [3] have proposed a unified criterion (we call it Ω-criterion) which represents the trade-off between energetic benefits and energetic losses due to irreversibilities for a specific job of any energy converter. On the other hand, studies on the energetics of Brownian motors, a thermal ratchet with a load, are relevant because of two main reasons [4,5]. First, due to the connection between the mechanical and thermal aspects of dynamics described by stochastic Langevin and Fokker-Planck equations. Second, in order to find guidelines for the design of efficient physical realizations.

Recently, we have studied the behavior of efficiency and power under some optimal operating regimes for irreversible cycle models of macroscopic heat engines [3] and for Feynman's ratchet and pawl engine and its electric counterpart, the diode engine [6]. In this paper we extend these studies to an isothermal rocking ratchet. In particular, we present a systematic analysis of the regimes of maximum work and efficiency and the results of efficiency and work when the ratchet is optimized under the Ω-criterion. Besides the concrete numerical results, we stress how efficiency and work in these isothermal models described by stochastic Langevin equations present facts qualitatively similar to those found in the nonisothermal energy converters previously analyzed [3,6]. The results of efficiency and work under the Ω-criterion are intermediate between those obtained under the maximum work and efficiency regimes. Such intermediate regime could be valuable for some real biological motors, which seem to be optimized from the velocity and the efficiency standpoints [7].

In the following section we present the model and the optimization regimes. In Sec. III we describe our numerical results, which are discussed in terms of the net current, input energy, and dissipated heat in Sec. IV. Finally we present a brief summary and some concluding remarks in Sec. V.

II. THE MODEL AND THE OPTIMIZATION CRITERIA

It is not our purpose here to describe the optimization of any existing model [8–14] of rocking ratchets accounting for nonadiabatic effects, asymmetric external forces, and inhomogeneous friction coefficients. Instead we will consider the simplest forced thermal ratchet in the adiabatic limit [12,13]. We assume an overdamped Brownian particle moving in an homogeneous ratchet potential under an external periodic force $F(t)$ at temperature T with a Langevin equation given by

$$\dot{x}=-\left[\frac{\partial V_0(x)}{\partial x}+\frac{\partial V_L(x)}{\partial x}\right]+F(t)+\xi(t), \tag{1}$$

where $\xi(t)$ is a Gaussian white noise with zero mean and $\langle\xi(t)\xi(t')\rangle=2kT\delta(t-t')$ (k is the Boltzmann constant); $V_0(x)$ is a periodic, asymmetric, and piecewise-linear potential given by

$$V_0(x)=\begin{cases}\dfrac{Q}{\lambda_1}x & 0<x\leqslant\lambda_1\\[2ex] \dfrac{\lambda_1+\lambda_2-x}{\lambda_2}Q & \lambda_1<x\leqslant\lambda_1+\lambda_2,\end{cases} \tag{2}$$

thus with intensity Q, spatial period $\lambda=\lambda_1+\lambda_2$, and a symmetry breaking amplitude $\Delta=\lambda_1-\lambda_2$; $V_L(x)$ is the potential due to the load, $\partial V_L(x)/\partial x=l>0$. When the external force $F(t)$, a square wave of amplitude A, is applied in either direction during a time much larger than any other time scale involved in the system (adiabatic limit) the solution of the quasistatic Fokker-Planck equation gives an induced current $J(A)$, which reads [9,12]

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$$J=\frac{P_2^2\sinh\left[\frac{\lambda(A-l)}{2kT}\right]}{kT\left[\frac{\lambda}{Q}\right]^2P_3-\frac{\lambda}{Q}P_1P_2\sinh\left[\frac{\lambda(A-l)}{2kT}\right]} \tag{3}$$

with

$$P_1=\Delta+\frac{(\lambda^2-\Delta^2)(A-l)}{4Q}, \tag{4}$$

$$P_2=\left[1-\frac{\Delta(A-l)}{2Q}\right]^2-\left[\frac{\lambda(A-l)}{2Q}\right]^2, \tag{5}$$

$$P_3=\cosh\left[\frac{2Q-\Delta(A-l)}{2kT}\right]-\cosh\left[\frac{\lambda(A-l)}{2kT}\right]. \tag{6}$$

The input energy (per unit time) from the external force to the ratchet is $E_{in}=A[J(A)-J(-A)]/2$, the work (per unit time) that the ratchet extracts from the external force is $W=l[J(A)+J(-A)]/2$, and the efficiency of the energy transformation is $\eta=W/E_{in}$ [12].

Regarding the optimization procedure we will analyze the two natural regimes of maximum work and efficiency. In addition, as noted in the introduction, we will consider a third performance regime which represents a compromise between work output and losses due to irreversibilities [3]. Usually, the analysis of irreversibilities in a real thermodynamic process needs the evaluation of the entropy generation, which is the basic magnitude of standard optimization criteria as entropy generation minimization and exergy analysis [15]. On the contrary, optimization with respect to Ω-function does not need the explicit evaluation of the entropy generation, a subtle and difficult task in many systems far from the equilibrium. Details on its derivation can be found in Ref. [3]. Mathematically the Ω-function for heat engines reads as $\Omega(y;\{\gamma\})=[2\eta(y;\{\gamma\})-\eta_{max}(\{\gamma\})]W(y;\{\gamma\})/\eta(y;\{\gamma\})$, where y denotes the appropriate independent variables, $\{\gamma\}$ denotes a set of parameters which can be considered as controls, $\eta(y;\{\gamma\})$ is the conventional efficiency, $W(y;\{\gamma\})$ is the work delivered, and $\eta_{max}(\{\gamma\})$ is the maximum possible value of the efficiency in the allowed range of values of y for given γ's [see Eq. (2) in Ref. [3]]. So, its implementation in any heat engine only needs the knowledge of the work and efficiency in terms of the variables and controls defining a given thermodynamic process.

III. NUMERICAL RESULTS

A. Deterministic limit

Let us begin with the case for which thermal noise is absent, i.e., when $T\to 0$. In this case the current density is given by [10,13]

$$J(A)=\begin{cases}0 & -\frac{Q}{\lambda_2}\leqslant A-l\leqslant\frac{Q}{\lambda_1}\\ \frac{1}{\lambda}\left[(A-l)-\frac{Q^2}{Q\Delta+\lambda_1\lambda_2(A-l)}\right] & \text{otherwise.}\end{cases} \tag{7}$$

In Fig. 1 we show 3D-plots of W and η for fixed values of the ratchet potential ($Q=1$, $\lambda=1$, $\lambda_1=0.8$, $\lambda_2=0.2$) using Eq. (7) for $J(A)$. Some 2D-plots versus A for given values of l are shown in Fig. 2. It is clear from these figures that W and η are functions which can be maximized with respect to both A and l. For a fixed value of l (A) each function presents a relative maximum for some $A(l)$. However, the absolute maximum values occur for a unique value of the couple (A,l). Similar qualitative plots are found at finite temperatures (see below). Thus, A and l can be considered as the appropriate independent variables of the optimization problem, while the temperature of the thermal bath and the parameters of the ratchet potential as the set of controls. Due to the strong nonlinearity of $J(A,l)$ out of the mobility gap, $-Q/\lambda_2\geqslant A-l\geqslant Q/\lambda_1$, to obtain an analytical solution of the problem for arbitrary changes of A and l is not an easy task, even in the deterministic limit. All reported results have been obtained numerically using the standard MATHEMATICA package.

The calculated values of the deterministic amplitude and load giving maximum efficiency, $A_{max\,\eta}$ and $l_{max\,\eta}$, respectively, and of the maximum efficiency $\eta_{max}\equiv\eta(A_{max\,\eta},l_{max\,\eta})$ agree with those already reported by Sokolov [13] in terms of the parameters of the ratchet potential: $A_{max\,\eta}=3.125=[Q(\lambda_1+\lambda_2)]/2\lambda_1\lambda_2$, $l_{max\,\eta}=1.875=[Q(\lambda_1-\lambda_2)]/2\lambda_1\lambda_2$, and $\eta_{max}=0.60=l_{max\,\eta}/A_{max\,\eta}=(\lambda_1-\lambda_2)/(\lambda_1+\lambda_2)=2\lambda_1/(\lambda_1+\lambda_2)-1$. The deterministic maximum work $W(A_{maxW},l_{maxW})\equiv W_{max}=1.04$ is achieved for slightly higher values of A and smaller values of l [$A_{maxW}=4.04$ and $l_{maxW}=0.95$] for which the efficiency is $\eta_{maxW}\equiv\eta(A_{maxW},l_{maxW})=0.235$. The deterministic maximum efficiency regime is not operative since it implies no work and the deterministic maximum work regime implies a drastic decreasing of the efficiency up to 0.235 from 0.6. Between these two regimes the Ω-criterion yields an efficiency $\eta_{max\Omega}\equiv\eta(A_{max\Omega},l_{max\Omega})$ approaching 0.44 (close to the maximum 0.6), while work, $W_{max\Omega}\equiv W(A_{max\Omega},l_{max\Omega})$, remains finite with a value close to 0.63 (above the half of the maximum work) but now the needed optima amplitude and load are $A_{max\Omega}=3.46$ and $l_{max\Omega}=1.54$, respectively. In Fig. 2 it can be seen how η and W behave versus A for some l values. Note, in particular, how

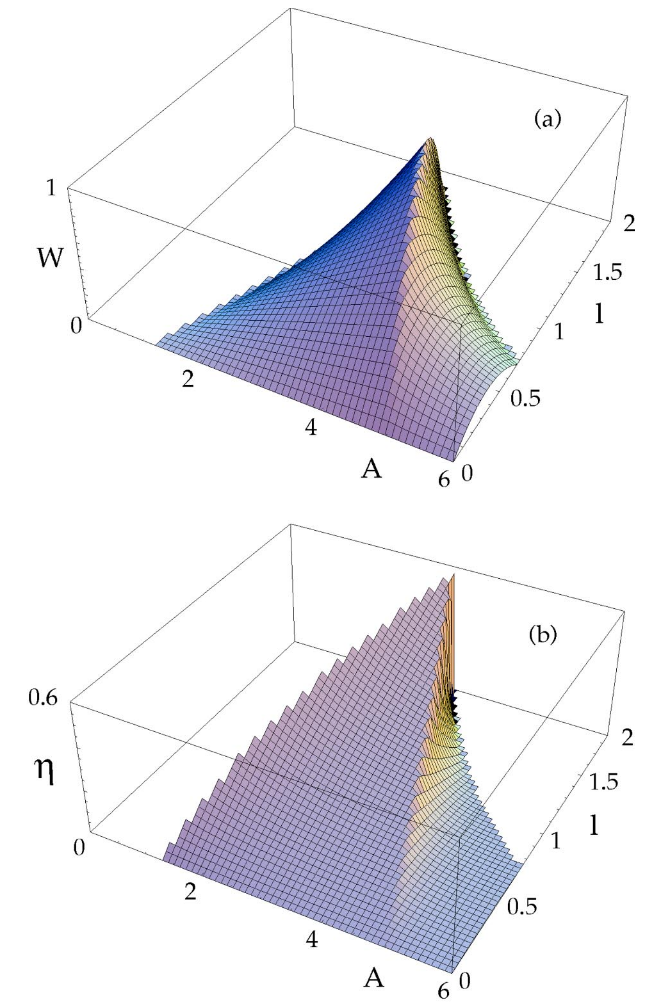

FIG. 1. Work (a) and efficiency (b) vs A and l in the deterministic limit, $kT=0$, for the ratchet parameters $Q=1$, $\lambda=1$, $\lambda_1=0.8$, $\lambda_2=0.2$. The plotted loads and amplitudes are those for which the ratchet is able to extract energy from the rocking mechanism.

as the maximum efficiency is reached [$A\rightarrow 3.125$ and $l\rightarrow 1.875$], work also approaches zero.

B. Finite temperatures

At finite temperatures all the results have been obtained using expression (3) for the current density. As an illustration, we show 3D-plots of η and W at $kT=0.1$ in Fig. 3. It is clear again that both A and l are appropriate independent variables also at finite temperatures. The calculated optimized values of efficiency, work, external amplitude, and load are plotted in Fig. 4 versus the bath temperature. These figures clearly show that as kT increases, the maximum work and work under maximum Ω-conditions decrease in a monotonic way, see Fig. 4(a). The same behavior is observed for the efficiencies under the three regimes [Fig. 4(b)]. Exception to these regular behaviors is the work output under maximum efficiency, $W_{max\,\eta}$: it shows a quite different, nonmonotonic behavior with a clear maximum at $kT\approx 0.1$. Note that at finite temperatures, Ω gives also efficiencies and

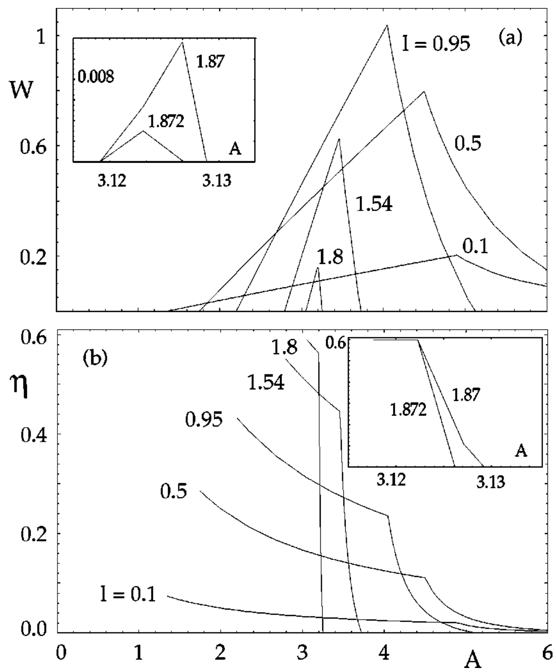

FIG. 2. Deterministic numerical results for the work (a) and efficiency (b) in terms of A for the labeled values of the load l. In particular, $l=0.95$ is the optimized load at maximum work conditions and $l=1.54$ at maximum Ω conditions. The insets shows the behavior close to maximum efficiency conditions, $l=1.875$ (see text).

works which are intermediate between those predicted by the maximum efficiency and maximum work regimes.

For the optimized amplitudes and loads we observe a distinct behavior on the temperature, while the amplitudes first decrease up to $kT\approx 0.15$ but above this temperature they strongly increase, see Fig. 4(c), the optimized loads decrease continuously as temperature increases in the three regimes, see Fig. 4(d). Also note that while $A_{max\Omega}$ is intermediate between $A_{max\,\eta}$ and A_{maxW} at each temperature [$A_{max\,\eta}<A_{max\Omega}<A_{maxW}$], the optimized load $l_{max\Omega}$ is intermediate between $l_{max\,\eta}$ and l_{maxW} only at very low temperatures and at $kT\gtrsim 0.1$. Moreover, at $kT\gtrsim 0.1$ we obtain $l_{maxW}>l_{max\,\eta}$, while at very low temperatures $l_{max\,\eta}>l_{maxW}$ in opposition with the behavior of the amplitudes.

At high enough temperatures the ratchet effect tends to disappear and the three regimes give practically null efficiency and null work output. The progressive larger amplitudes needed to move progressive smaller loads make the ratchet quite inefficient due to the progressive importance of the thermal noise against fluctuation effects.

IV. NET CURRENT, INPUT ENERGY, AND DISSIPATED HEAT

Because the system under study is in contact with just one thermal bath at temperature T, the energetics of the system can be easily expressed in terms of the entropy production or dissipated heat Q_{dis} in the thermal bath at temperature T and the input energy E_{in}. So, in this particular system the efficiency is $\eta=1-(Q_{dis}/E_{in})$ and work becomes $W=E_{in}$

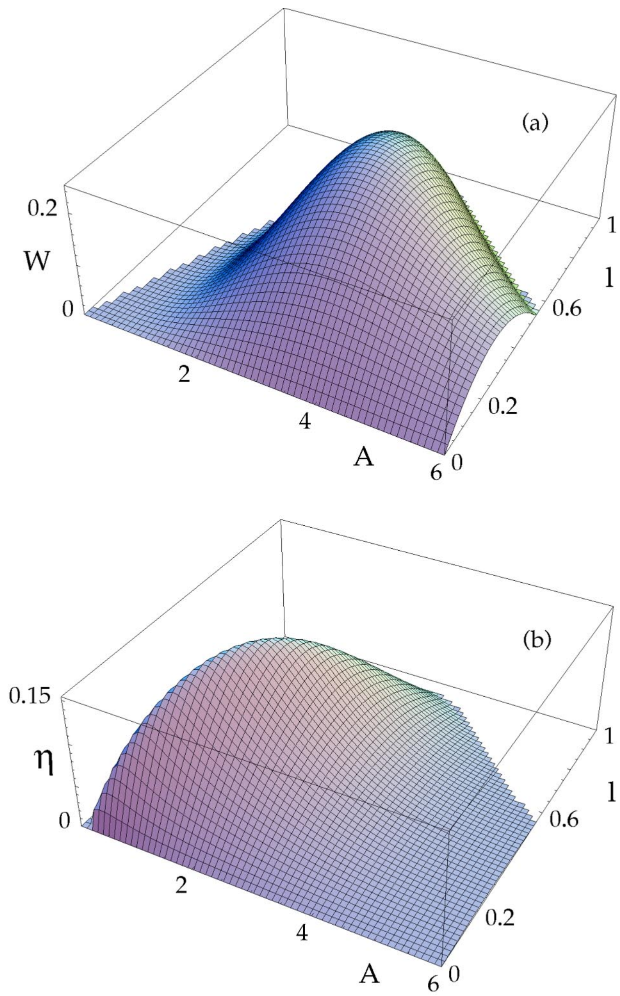

FIG. 3. As in Fig. 1 but at $kT=0.1$.

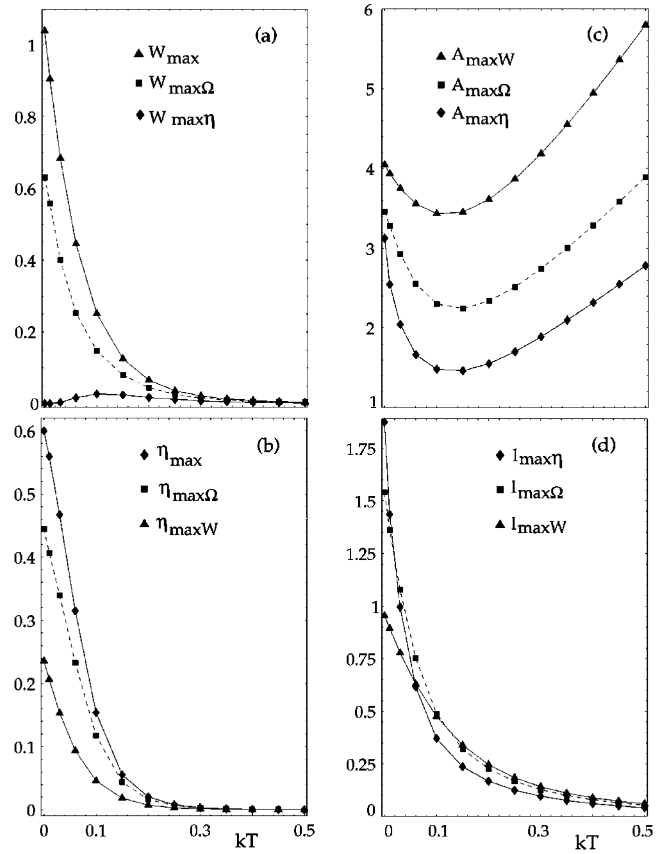

FIG. 4. Numerical values of the optimized works (a) efficiencies (b), external amplitudes (c), and loads (d) vs kT.

$-Q_{dis}$, where Q_{dis}/T is the total entropy production per particle since the system can be considered in a stationary state with constant entropy [5,14]. So, the behaviors of the optimized efficiencies and works in Figs. 4(a) and 4(b) can be properly explained by analyzing the kT evolution of E_{in} and Q_{dis}. However, first, we provide an intuitive explanation of the behaviors of optimized amplitudes and loads in Figs. 4(c) and 4(d) starting from the average current $\langle J\rangle=[J(A,l)+J(-A,l)]/2$.

In Fig. 5 we plot $\langle J\rangle$ versus A for some l values at three representative temperatures: $kT=0$, 0.1, and 0.4. Under deterministic conditions, see Fig. 5(a), each l-line in $\langle J\rangle$ begins its positive slope [coming from the current $J(A,l)$] at an amplitude given by the mobility gap $A=Q/\lambda_1+l$ and peaks at $A=Q/\lambda_2-l$. At the right of each peak the inverse current $J(-A,l)$ provokes the negative slope of each line. Accordingly, as l increases the corresponding maximum shifts to the left and above some l-value $\langle J\rangle$ becomes negative at any A. Maximum work, proportional to maximum $\langle J\rangle$, should be thus achieved at some load not too small [in order to keep $J(A,l)$ as high as possible] nor too large [in order to avoid intense negative $J(-A,l)$ values] and should be located at some A into the mobility gap. In our case, numerical calculations give $l_{maxW}=0.95$ and $A_{maxW}=4.04$. As temperature increases, Fig. 5(b), two factors play a role: the mobility gaps shift to the left and the reversal current $J(-A,l)$ progressively becomes stronger even at low loads. Thus $\langle J\rangle$ remains positive at lower loads for which the maxima are clearly shifted to smaller amplitudes. With a further increasing of temperature, Fig. 5(c), small positive net current is found only at very low loads peaking at high A-values. From all the above we conclude that the (expected) monotonic kT decreasing of l_{maxW} and the (perhaps, unexpected) nonmonotonic behavior of A_{maxW} are the joint consequence of avoiding the progressive reversal current as temperature increases and of the shifting to the left of the mobility gap.

The deterministic maximum efficiency is achieved at values of A and l corresponding to a stagnation situation when the average current just vanishes [13]. This is, when $Q/\lambda_1+l=Q/\lambda_2-l$ and A is equal to one-half of the gap width [in our case $l_{max\eta}=Q/2[(1/\lambda_2)-(1/\lambda_1)]=1.875$ and $A_{max\eta}=3.125$]. As temperature increases the location of the maximum efficiency, mainly imposed by that of maximum work, is also influenced by the evolution of input energy. Since the minimum (and null) value of each $E_{in}(A,l)$ moves to smaller amplitudes and loads as temperature increases, Figs. 5(a) and 5(b) each $A_{max\eta}$ should go below the corresponding A_{maxW} and each $l_{max\eta}$ should decay faster than the corresponding l_{maxW}. This last effect disappears at enough high temperatures because the almost null dependence of E_{in} on load, Fig. 5(c), and thus we find that $l_{max\eta}\to l_{maxW}$ at these temperatures.

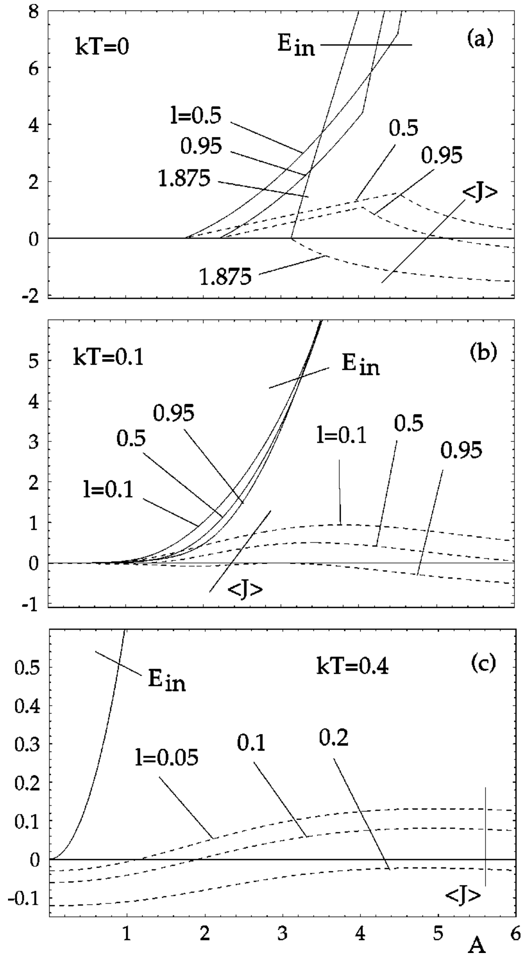

FIG. 5. Input energy E_{in} (solid lines) and net current $\langle J\rangle$ (dashed lines) vs A at $kT=0$ (a), 0.1 (b), and 0.4 (c) for the labeled l values. In (c) the different $E_{in}(l)$ values are indistinguishable. Note also the different scales of the vertical axes in the three figures.

The Ω-function can be expressed as $\Omega=[2\eta-\eta_{max}]W/\eta=[2\eta-\eta_{max}]E_{in}$. So, the location of its maxima mainly depends on the maxima of both η and E_{in}. Because the monotonic growth of E_{in} with A at any l, Fig. 5, the first consequence is that, at given temperature, each $A_{max\Omega}$ is greater than the corresponding $A_{max\eta}$. The l dependence in Ω is a little bit subtle: at the (low) temperatures for which $A_{max\Omega}$ decreases, the loads giving maximum input energy become smaller, Figs. 5(a) and 5(b), but as kT increases $A_{max\Omega}$ also increases and the maximum input energy is achieved at higher loads. We think that this nonregular dependence of $E_{in}(A,l)$ could be the origin of the intriguing decaying of $l_{max\Omega}$ with respect to those of $l_{max\eta}$ and l_{maxW} in Fig. 4(d). Nevertheless, we stress that the nonintermediate value of $l_{max\Omega}$ in a particular range of low temperatures does not invalidate the Ω-criterion as an optimum operating regime. In FTT, any regime giving efficiency over the efficiency at maximum power and a power over the power at maximum efficiency is said to be optimal [1,2]. In this line the Ω-criterion is an optimum operating regime in the adiabatic rocking system under study, independently of the concrete behaviors of the independent variable.

Figure 6 shows Q_{dis} and E_{in} versus l at $kT=0$ and 0.1 for the A values giving maximum in each regime. We also plot in each case the corresponding efficiency and work. We observe in Figs. 6(a) and 6(d) that at the maximum work regime the optimized (A,l)-values at each temperature are, as expected, those giving maximum $E_{in}-Q_{dis}$ and minimum values of both E_{in} and Q_{dis}. Thus, at each temperature the optimized l_{maxW} and A_{maxW} values are the required conditions to a ratchet performance with (relative) minima for the input energy and dissipated heat, their difference being the maximum possible. In the maximum efficiency regime, Figs. 6(c) and 6(f), $l_{max\eta}$ and $A_{max\eta}$ are the needed values in order to get the minimum (Q_{dis}/E_{in})-quotient at any temperature. Only at $kT=0$, Fig. 6(c), the minimum of (Q_{dis}/E_{in}) coincides with the (absolute) minima of both Q_{dis} and E_{in}: maximum efficiency is achieved when $Q_{dis}\to 0$ and $E_{in}\to 0$ with $Q_{dis}/E_{in}\to 0.4$ ($\eta_{max}=1-0.4=0.6$) but $E_{in}-Q_{dis}\to 0$, thus yielding the null value of work under maximum efficiency conditions in the deterministic mode of operation. The maximum Ω-regime implies relative minima for both E_{in} and Q_{dis} under deterministic conditions, Fig. 6(b), but this is not true at finite temperatures, Fig. 6(e).

The results in Fig. 6 show that only the maximum work regime implies (relative) minimum values of input energy and dissipated heat at any temperature, while the absolute minima of these magnitudes are only achieved by the deterministic efficiency regime. Some remarks on the entropy production in the bath, (Q_{dis}/T), are in order at this point. For every value of A, there is a value of l that minimizes the entropy production at given temperature. At small temperatures the loads minimizing entropy decrease as the amplitude increases and above some A this load tends to zero. At high enough temperatures the entropy production is a growing function of A with a relative minimum at $l\to 0$. From the results for Q_{dis} in Figs. 6(a)–6(c) it is clear that at deterministic conditions the states of maximum W, Ω, and η correspond with states giving local minimum values of the entropy surface. However at finite temperatures this is only true for the states under the maximum work regime.

We show in Fig. 7 a more detailed kT-evolution of E_{in} and Q_{dis} in each regime. It is clear that both magnitudes increase as temperature increases but at a fixed temperature they are greater in the maximum work regime than under maximum efficiency conditions with, as expected, intermediate values in the Ω-criterion. So, from the point of view of the needed input energy and the unavoidable dissipated heat, the maximum work is a quite unfavorable operating regime in the adiabatic rocking ratchet at any temperature in opposition to the maximum efficiency regime. From Fig. 7 it is straightforward to understand the calculated monotonic decreasing with temperature of the optimized efficiencies, $1-(Q_{dis}/E_{in})$, in Fig. 4(b), and works, $E_{in}-Q_{dis}$, in Fig. 4(a). In particular, the behavior of the work at maximum efficiency conditions follows directly from Fig. 7(b): as noted above $E_{in}-Q_{dis}\to 0$ at $kT=0$, thus yielding the null value of work under maximum efficiency conditions in the deterministic mode of operation, but as kT increases the difference $E_{in}-Q_{dis}=W$ first increases and then decreases with

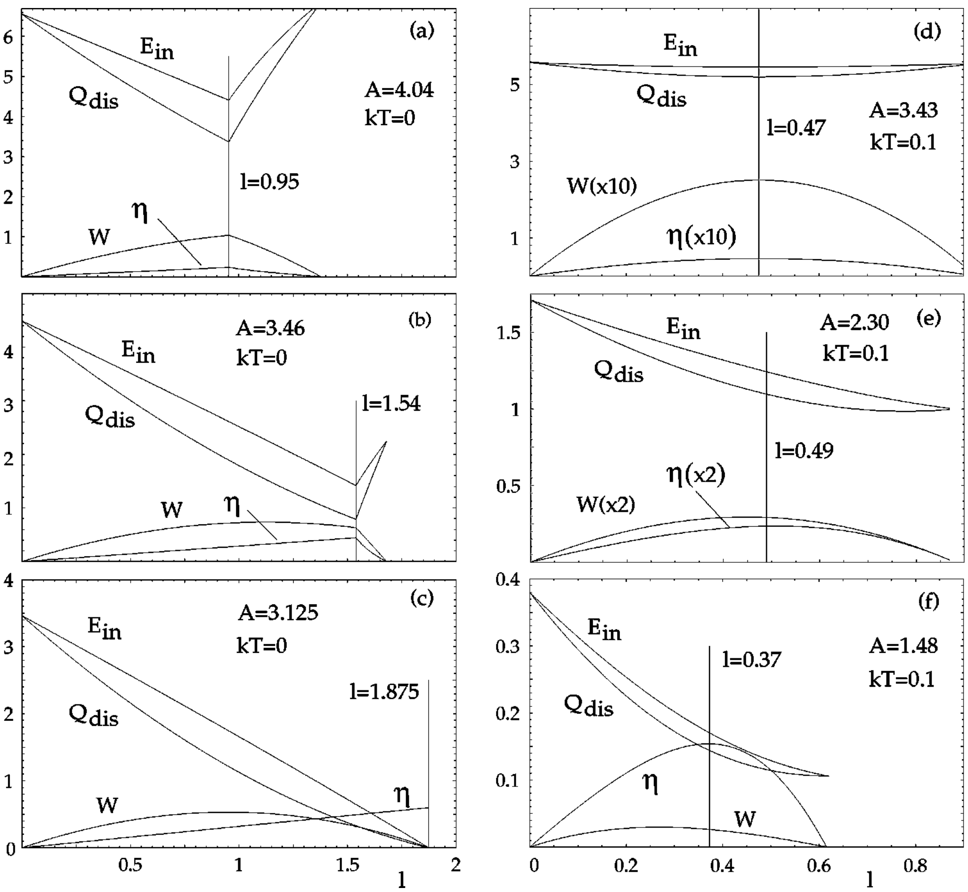

FIG. 6. Input energy and dissipated heat vs load at the labeled A values at $kT=0$ and 0.1. (a) and (d) apply for the maximum work regime, (b) and (e) for the maximum Ω regime, and (c) and (f) for the maximum efficiency regime. The labeled A and l are the values of the independent variables giving the maximum of each regime at the considered temperature.

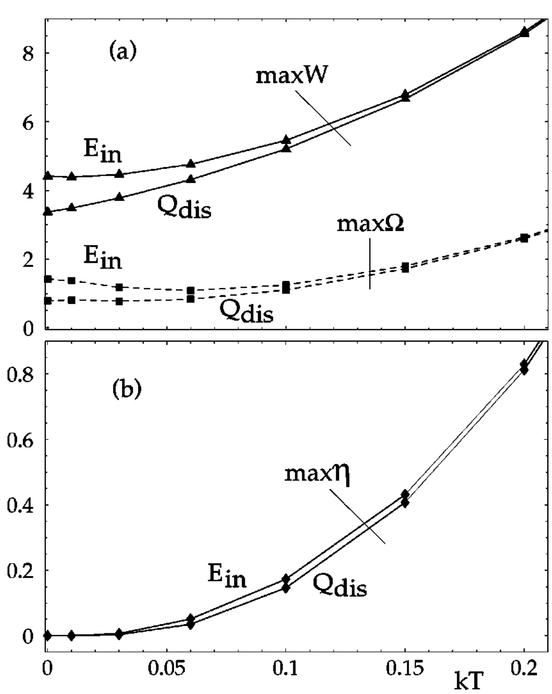

FIG. 7. kT evolution of input energy and dissipated heat under conditions of maximum work and maximum Ω (a) and maximum efficiency (b).

a maximum value around $kT=0.1$, which is the peak observed in Fig. 4(a).

V. SUMMARY AND CONCLUDING REMARKS

We have presented a systematic study of the efficiency and work for an adiabatic rocking ratchet with symmetric external amplitudes when optimized under three different regimes: maximum work, maximum efficiency, and one more which represent the trade-off between work output and losses by irreversibilities. Beyond particular numerical results we stress the following main points.

(a) The existence of very concrete external amplitudes and loads for which both efficiency and work achieve their maximum possible values (quite significant at low temperatures) for given ratchet potential and temperature of the thermal bath.

(b) The efficiency and work of the ratchet when optimized with the Ω-criterion present values located between those given by the maximum efficiency and maximum work regimes. To achieve this the Ω-performance consumes intermediate input energy and dissipates an intermediate amount of heat; and

(c) the nonmonotonic decreasing with temperature of the optimized work under maximum efficiency conditions and of the external amplitude in the three regimes, in opposition to the monotonic behavior observed for the three efficiencies, maximum work, work under maximum Ω conditions, and load in the three regimes.

The analysis presented here can be extended to other more complete models of rocked ratchets [8–11,13] and other types of Brownian motors [4,16]. In particular, for nonadiabatic rocked ratchets [8] incorporating additional time and/or spatial asymmetries [10] and inhomogeneous friction coefficients [11], the appearance of current reversals and the fact that the efficiency can be maximized at finite temperature make an analysis on optimal operating regimes specially relevant.

It has been reported [7] that some protein motors seem to be optimized from the velocity and the efficiency standpoints. To explain the observed behavior of this kind of motors an alternative definition of efficiency [17] incorporating the work done by friction forces has been proposed. As it has been pointed out by Parrondo *et al.* [14] that one has to be careful with this type of definitions, since the work done against the friction is always dissipated as heat to the thermal bath. If the task of these molecular motors is to do useful work against an external force and besides to achieve some velocity, perhaps they can be properly analyzed under a performance regime representing a compromise between efficiency and power. In fact, not only in physics and biology but also in several spheres of human activity compromise has been proposed [18] as a major unifying thread. Some biological systems described by linear irreversible thermodynamics have been worked out under this point of view [19] and an ecological-like optimization criterion, very similar to Ω but including the explicit evaluation of the entropy generation, has been applied [20].

It is interesting to face the obtained results for the isothermal, rocking ratchets to those reported in FTT for macroscopic heat engines [3] and for the mechanical and electric rectifiers [6]. All these systems show as a significant characteristic that the maximum power and maximum efficiency regimes are close but noncoincident. In other words, both power and efficiency show a maximum for different, but close, values of some appropriate independent variable: the pressure ratio for Joule-Brayton cycles, the compression ratio for automotive cycles, the temperature ratio for irreversible Carnotlike cycles, the potential energy of the external weight in Feynman's ratchet, and the current in the diode engines. This fact gives rise to a looplike behavior for power-efficiency plots, which is a specific sign of real motors [1,2]. The results shown in Figs. 1 and 3 corroborate the above requirements for the isothermal rocking ratchet.

In summary, heat engines seem to show some similarities for the efficiency and work (or power) when studied in terms of appropriate independent variables, independent of their nature and size. An unified analysis under different optimal performance regimes could give guidelines in order to design efficient motors and to compare their operation in different situations.

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